7.5 Exercises

1–82 Evaluate the integral.

1. $\int \cos x\,(1 + \sin^2 x)\,dx$
2. $\int_0^1 (3x + 1)^{\sqrt{2}}\,dx$
3. $\int \dfrac{\sin x + \sec x}{\tan x}\,dx$
4. $\int \dfrac{\sin^3 x}{\cos x}\,dx$
5. $\int \dfrac{t}{t^4 + 2}\,dt$
6. $\int_0^1 \dfrac{x}{(2x + 1)^3}\,dx$
7. $\int_{-1}^1 \dfrac{e^{\arctan y}}{1 + y^2}\,dy$
8. $\int t \sin t \cos t\,dt$
9. $\int_1^3 r^4 \ln r\,dr$
10. $\int_0^4 \dfrac{x - 1}{x^2 - 4x - 5}\,dx$
11. $\int \dfrac{x - 1}{x^2 - 4x + 5}\,dx$
12. $\int \dfrac{x}{x^4 + x^2 + 1}\,dx$
13. $\int \sin^5 t \cos^4 t\,dt$
14. $\int \dfrac{x^3}{\sqrt{1 + x^2}}\,dx$
15. $\int \dfrac{dx}{(1 - x^2)^{3/2}}$
16. $\int_0^{\sqrt{2}/2} \dfrac{x^2}{\sqrt{1 - x^2}}\,dx$
17. $\int_0^{\pi} t \cos^2 t\,dt$
18. $\int_1^4 \dfrac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$
19. $\int e^{x + e^x}\,dx$
20. $\int e^{\sqrt{x}}\,dx$
21. $\int \arctan \sqrt{x}\,dx$
22. $\int \dfrac{\ln x}{x\sqrt{1 + (\ln x)^2}}\,dx$
23. $\int_0^1 (1 + \sqrt{x})^8\,dx$
24. $\int_0^4 \dfrac{6z + 5}{2z + 1}\,dz$
25. $\int \dfrac{3x^2 - 2}{x^2 - 2x - 8}\,dx$
26. $\int \dfrac{3x^2 - 2}{x^3 - 2x - 8}\,dx$
27. $\int \dfrac{dx}{1 + e^x}$
28. $\int \sin \sqrt{at}\,dt$
29. $\int \ln\left(x + \sqrt{x^2 - 1}\right)\,dx$
30. $\int_{-1}^2 |\,e^x - 1\,|\,dx$
31. $\int \sqrt{\dfrac{1 + x}{1 - x}}\,dx$
32. $\int \dfrac{\sqrt{2x - 1}}{2x + 3}\,dx$
33. $\int \sqrt{3 - 2x - x^2}\,dx$
34. $\int_{\pi/4}^{\pi/2} \dfrac{1 + 4\cot x}{4 - \cot x}\,dx$
35. $\int \cos 2x \cos 6x\,dx$
36. $\int_{-\pi/4}^{\pi/4} \dfrac{x^2 \tan x}{1 + \cos^4 x}\,dx$
37. $\int_0^{\pi/4} \tan^3\theta \sec^2\theta\,d\theta$
38. $\int_{\pi/6}^{\pi/3} \dfrac{\sin\theta \cot\theta}{\sec\theta}\,d\theta$
39. $\int \dfrac{\sec\theta \tan\theta}{\sec^2\theta - \sec\theta}\,d\theta$
40. $\int \dfrac{1}{\sqrt{4y^2 - 4y - 3}}\,dy$
41. $\int \theta \tan^2\theta\,d\theta$
42. $\int \dfrac{\tan^{-1} x}{x^2}\,dx$
43. $\int \dfrac{\sqrt{x}}{1 + x^3}\,dx$
44. $\int \sqrt{1 + e^x}\,dx$
45. $\int x^5 e^{-x^3}\,dx$
46. $\int \dfrac{(x - 1)e^x}{x^2}\,dx$
47. $\int x^3 (x - 1)^{-4}\,dx$
48. $\int_0^1 x\sqrt{2 - \sqrt{1 - x^2}}\,dx$
49. $\int \dfrac{1}{x\sqrt{4x + 1}}\,dx$
50. $\int \dfrac{1}{x^2\sqrt{4x + 1}}\,dx$
51. $\int \dfrac{1}{x\sqrt{4x^2 + 1}}\,dx$
52. $\int \dfrac{dx}{x(x^4 + 1)}$
53. $\int x^2 \sinh mx\,dx$
54. $\int (x + \sin x)^2\,dx$
55. $\int \dfrac{dx}{x + x\sqrt{x}}$
56. $\int \dfrac{dx}{\sqrt{x} + x\sqrt{x}}$
57. $\int x\sqrt[3]{x + c}\,dx$
58. $\int \dfrac{x \ln x}{\sqrt{x^2 - 1}}\,dx$
59. $\int \cos x \cos^3(\sin x)\,dx$
60. $\int \dfrac{dx}{x^2\sqrt{4x^2 - 1}}$
61. $\int \dfrac{d\theta}{1 + \cos\theta}$
62. $\int \dfrac{d\theta}{1 + \cos^2\theta}$
63. $\int \sqrt{x}\, e^{\sqrt{x}}\,dx$
64. $\int \dfrac{1}{\sqrt{\sqrt{x} + 1}}\,dx$
65. $\int \dfrac{\sin 2x}{1 + \cos^4 x}\,dx$
66. $\int_{\pi/4}^{\pi/3} \dfrac{\ln(\tan x)}{\sin x \cos x}\,dx$
67. $\int \dfrac{1}{\sqrt{x + 1} + \sqrt{x}}\,dx$
68. $\int \dfrac{x^2}{x^6 + 3x^3 + 2}\,dx$
69. $\int_1^{\sqrt{3}} \dfrac{\sqrt{1 + x^2}}{x^2}\,dx$
70. $\int \dfrac{1}{1 + 2e^x - e^{-x}}\,dx$
71. $\int \dfrac{e^{2x}}{1 + e^x}\,dx$
72. $\int \dfrac{\ln(x + 1)}{x^2}\,dx$
73. $\int \dfrac{x + \arcsin x}{\sqrt{1 - x^2}}\,dx$
74. $\int \dfrac{4^x + 10^x}{2^x}\,dx$
75. $\int \dfrac{1}{(x - 2)(x^2 + 4)}\,dx$
76. $\int \dfrac{dx}{\sqrt{x}(2 + \sqrt{x})^4}$
77. $\int \dfrac{xe^x}{\sqrt{1 + e^x}}\,dx$
78. $\int \dfrac{1 + \sin x}{1 - \sin x}\,dx$
79. $\int x \sin^2 x \cos x\,dx$
80. $\int \dfrac{\sec x \cos 2x}{\sin x + \sec x}\,dx$
81. $\int \sqrt{1 - \sin x}\,dx$
82. $\int \dfrac{\sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$

83. The functions $y = e^{x^2}$ and $y = x^2 e^{x^2}$ don't have elementary antiderivatives, but $y = (2x^2 + 1)e^{x^2}$ does. Evaluate $\int (2x^2 + 1)e^{x^2}\,dx$.

84. We know that $F(x) = \int_0^x e^{t^2}\,dt$ is a continuous function by FTC1, though it is not an elementary function. The functions

$$\int \frac{e^x}{x}\,dx \quad \text{and} \quad \int \frac{1}{\ln x}\,dx$$

are not elementary either, but they can be expressed in terms of F. Evaluate the following integrals in terms of F.

(a) $\int_1^2 \dfrac{e^x}{x}\,dx$ (b) $\int_2^3 \dfrac{1}{\ln x}\,dx$

85. $\int \sqrt{4x^2 - 4x - 3}\,dx$ 86. $\int \csc^5 x\,dx$

87. $\int \cos x \sqrt{4 + \sin^2 x}\,dx$ 88. $\int \dfrac{\cot x}{\sqrt{1 + 2\sin x}}\,dx$

* $\int \sqrt{\tan x}\,dx$

* $\int \dfrac{1}{x^7 - x}\,dx$

* $\int_0^1 \left(\sqrt[3]{1 - x^7} - \sqrt[7]{1 - x^3}\right)\,dx$

Exercises

Note: Additional practice in techniques of integration is provided in Exercises 7.5.

1–40 Evaluate the integral.

1. $\int_1^2 \frac{(x+1)^2}{x}\,dx$
2. $\int_1^2 \frac{x}{(x+1)^2}\,dx$
3. $\int_0^{\pi/2} \sin\theta\, e^{\cos\theta}\,d\theta$
4. $\int_0^{\pi/6} t \sin 2t\,dt$
5. $\int \frac{dt}{2t^2+3t+1}$
6. $\int_1^2 x^5 \ln x\,dx$
7. $\int_0^{\pi/2} \sin^3\theta \cos^2\theta\,d\theta$
8. $\int \frac{dx}{\sqrt{e^x-1}}$
9. $\int \frac{\sin(\ln t)}{t}\,dt$
10. $\int_0^1 \frac{\sqrt{\arctan x}}{1+x^2}\,dx$
11. $\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$
12. $\int \frac{e^{2x}}{1+e^{4x}}\,dx$
13. $\int e^{\sqrt[3]{x}}\,dx$
14. $\int \frac{x^2+2}{x+2}\,dx$
15. $\int \frac{x-1}{x^2+2x}\,dx$
16. $\int \frac{\sec^6\theta}{\tan^2\theta}\,d\theta$
17. $\int x \sec x \tan x\,dx$
18. $\int \frac{x^2+8x-3}{x^3+3x^2}\,dx$
19. $\int \frac{x+1}{9x^2+6x+5}\,dx$
20. $\int \tan^5\theta \sec^3\theta\,d\theta$
21. $\int \frac{dx}{\sqrt{x^2-4x}}$
22. $\int te^{\sqrt{t}}\,dt$
23. $\int \frac{dx}{x\sqrt{x^2+1}}$
24. $\int e^x \cos x\,dx$
25. $\int \frac{3x^3-x^2+6x-4}{(x^2+1)(x^2+2)}\,dx$
26. $\int x \sin x \cos x\,dx$
27. $\int_0^{\pi/2} \cos^3 x \sin 2x\,dx$
28. $\int \frac{\sqrt[3]{x}+1}{\sqrt[3]{x}-1}\,dx$
29. $\int_{-3}^3 \frac{x}{1+|x|}\,dx$
30. $\int \frac{dx}{e^x\sqrt{1-e^{-2x}}}$
31. $\int_0^{\ln 10} \frac{e^x\sqrt{e^x-1}}{e^x+8}\,dx$
32. $\int_0^{\pi/4} \frac{x \sin x}{\cos^3 x}\,dx$
33. $\int \frac{x^2}{(4-x^2)^{3/2}}\,dx$
34. $\int (\arcsin x)^2\,dx$
35. $\int \frac{1}{\sqrt{x+x^{3/2}}}\,dx$
36. $\int \frac{1-\tan\theta}{1+\tan\theta}\,d\theta$
37. $\int (\cos x + \sin x)^2 \cos 2x\,dx$
38. $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx$
39. $\int_0^{1/2} \frac{xe^{2x}}{(1+2x)^2}\,dx$
40. $\int_{\pi/4}^{\pi/3} \frac{\sqrt{\tan\theta}}{\sin 2\theta}\,d\theta$

41–55 Evaluate the integral or show that it is divergent.

41. $\int_1^\infty \frac{1}{(2x+1)^3}\,dx$
42. $\int_1^\infty \frac{\ln x}{x^4}\,dx$
43. $\int_2^\infty \frac{dx}{x \ln x}$
44. $\int_2^6 \frac{y}{\sqrt{y-2}}\,dy$
45. $\int_0^4 \frac{\ln x}{\sqrt{x}}\,dx$
46. $\int_0^1 \frac{1}{2-3x}\,dx$
47. $\int_0^1 \frac{x-1}{\sqrt{x}}\,dx$
48. $\int_{-1}^1 \frac{dx}{x^2-2x}$
49. $\int_{-\infty}^\infty \frac{dx}{4x^2+4x+5}$
50. $\int_1^\infty \frac{\tan^{-1}x}{x^2}\,dx$
51. $\int_2^\infty y \cdot e^{-3y}\,dy$
52. $\int_0^1 \frac{e^{1/x}}{x^3}\,dx$
53. $\int_0^2 x^2 \ln x\,dx$
54. $\int_0^\infty \frac{x \cdot \arctan x}{(1+x^2)^2}\,dx$
55. $\int_{\pi/2}^\pi \csc x\,dx$

56–60 Use the Comparison Theorem to determine whether the integral is convergent or divergent.

56. $\int_0^\infty \frac{\arctan x}{2+e^x}\,dx$
57. $\int_2^\infty \frac{x^2+x}{x^7+4x+3}\,dx$
58. $\int_0^\pi \frac{\sin^2 x}{\sqrt{x}}\,dx$
59. $\int_1^\infty \frac{2+e^{-x}}{x}\,dx$
60. $\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}\,dx$